

Charge dependence of $NN \rightarrow d\pi$

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Abstract

We calculate the isospin symmetry violating effects to the reactions $pp \rightarrow d\pi^+$ and $np \rightarrow d\pi^0$ arising from the different hadron masses and from the Coulomb interaction between the positive pion and the deuteron. These effects are large enough in the cross section and analyzing power A_y that they should be taken into account in comparisons of accurate experiments in different charge channels.

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Isospin symmetry has been used with great success to relate the experimentally available pion production reactions

$$pp \rightarrow d\pi^+ \quad (1)$$

and

$$np \rightarrow d\pi^0. \quad (2)$$

Another charge channel is $nn \rightarrow d\pi^-$, but this is impracticable. Further, positive (or negative) pion absorption on the deuteron

$$\pi^+d \rightarrow pp \quad (3)$$

can be related to (1) by detailed balance. Because the isospin one component of the np initial state has the probability $\frac{1}{2}$, then according to this symmetry the cross section for the reaction (2) is simply one half of the pp initiated reaction (1). Relative angular distributions and spin observables of both reactions should be the same.

However, the isospin symmetry is not exact. Since the masses of the initial state nucleons as well as the masses of the charged and neutral pions are different, the $T_z = 0$ reaction (2) has a 5.9 MeV lower threshold (CMS) than the $T_z = 1$ reaction (1). This has a potential to be a significant effect as compared with errors of modern accurate

experiments. Some of this difference can be accounted for by different phase space factors in the cross sections. In relative observables these factors cancel.

After the removal of the trivial phase space factors, nevertheless, there remain some charge dependent effects in the transition amplitudes themselves. These have three basic origins:

1) Kinematic difference: The relation of the initial momentum to the final momentum is not exactly the same for different reactions. Even for the same final momentum (the more relevant in threshold reactions with a large negative Q-value), the initial momentum in the baryon wave function is slightly different. The latter enters the transition matrix via the NN wave function in a way dependent on the partial wave and can, in principle, influence also the relative observables.

2) The Coulomb interaction enters the channel (1) in both the initial and the final state, while it is absent in (2). Presumably for the high energy nucleons it can be neglected, but it can cause a significant correction in the $d\pi^+$ final state in the threshold region. One may safely neglect the magnetic interaction (although it may be relatively important in the small *charge symmetry breaking* in $np \rightarrow d\pi^0$).

3) Charge dependent strong force: A more interesting possibility is charge dependence in dynamics, i.e. the interaction involved. The lighter and longer-ranged π^0 exchange enters differently in the pp and np initial states. This gives rise to an isotensor force both in NN scattering [1] and in the $NN \rightarrow \Delta N$ transition potential [2] as well as in pion s -wave rescattering. Another source for a charge dependent dynamic force can be meson mixings $\pi^0 \leftrightarrow \eta$ and $\rho^0 \leftrightarrow \omega$. However, the latter violate also charge symmetry and are presumably smaller.

The aim of this Letter is to study the first two contributions to charge dependence in pion production. The motivation is an urgent need for an estimate because of experimental reasons. The accuracy of experiments on $np \rightarrow d\pi^0$ is aiming at a level where even the possible variation from the above sources may be significant in comparing different charge channels [3]. Precision measurements of np scattering at intermediate energies often use pion production and isospin symmetry in normalizing the luminosity and the above differences could be a source of a systematic error. This in turn may have implications e.g. to the determination of the (charged) pion coupling to nucleons, a topic of much controversy recently [4]. Also there is progress in threshold experiments, where the Coulomb corrections are most relevant [5, 6, 7, 8]. In other reactions, such as $pp \rightarrow pp\pi^0$ its inclusion has been essential [9]. Therefore it is necessary, if not interesting, to get a quantitative estimate of charge dependence in the $NN \rightarrow d\pi$ reactions. Although these violations of the isospin symmetry may be small, to our knowledge there have not been any systematic calculations of these to really raise comparisons of experiments above *ad hoc* reliance on charge independence.

We shall discuss the charge dependent effects in two parts, showing the change in the observables most likely to be compared (the differential cross section and the analyzing power A_y). First we calculate the kinematic effect due to the change of the initial state making all comparisons for the same final pion momentum in the CMS. Then

we proceed to incorporate the final state Coulomb corrections in different approximate ways. The first one is simply to apply the Coulomb penetration factors $|C_l|^2$, and in the second one we replace the plane wave pion in the matrix elements by Coulomb scattering wave functions. Finally, the effect of the finite charge distribution is also estimated. In principle, one should consider also the polarizability of the deuteron. However, we end up with the conclusion that this is most likely a small effect, much smaller than the already small ones that we include. Technically including the polarizability would be also much harder.

A detailed discussion of the model used can be found in Ref. [10]. In addition to the direct production from the initial NN states, we include by the coupled-channels method p -wave pion rescattering through the $\Delta(1232)$ isobar (dominant above 350 MeV in p -wave production from 1D_2) and also s -wave rescattering (dominant at threshold). The Reid soft core wave function is used for the deuteron. This model gives rather good predictions to the cross sections and spin observables through the Δ region, but overestimates threshold production by about 50 % [11].

In the naive quark model the mass difference between the NN channels and the ΔN channels does not depend on the charge [2], because the same six quarks are involved in both. This means that the kinetic energy required for the excitation of a ΔN intermediate state is the same for the np and pp initial states. Therefore no difference in the transition $NN \rightarrow \Delta N$ arises from the baryon masses at the same incident nucleon kinetic energy. It was also pointed out in Ref. [2] that in lowest order the internal Coulomb effects should have a similar effect to the total masses in all channels.

The production cross section of $pp \rightarrow d\pi^+$ in the centre-of-mass system can be expressed in terms of the matrix elements of the production operator as

$$\frac{d\sigma_{\text{prod}}}{d\Omega} = \frac{1}{(2\pi)^2} \frac{\omega_q E_d E_1 E_2 q}{ps} \frac{1}{4} \times \sum_{\mu SM} |\langle \psi_d^\mu | H^\pi | \phi^{SM} \rangle|^2 = P_{pp} \times R_{pp}. \quad (4)$$

Here q is the pion momentum and p the initial nucleon momentum. The energies E_1 and E_2 are the initial nucleon energies, while ω_q and E_d are the final state energies for the pion and deuteron, and s is the Mandelstam variable. All kinematic quantities refer to the centre-of-mass system. The cross section is explicitly factorized to the phase space factor P_{pp} and the sum of squared matrix elements R_{pp} . For the np reaction (2) one needs also the additional isospin factor $\frac{1}{2}$ relative to the reaction (1) and our factorization is $2\sigma(np) = P_{np} \cdot R_{np}$, so that also R_{np} corresponds to similar pure isospin one matrix elements as R_{pp} does. The notation is valid for both the differential and integrated cross sections. In the former the R is just angle dependent.

The absorption cross section (3)

$$\frac{d\sigma_{\text{abs}}}{d\Omega} = \frac{1}{(2\pi)^2} \frac{\omega_q E_d E_1 E_2 p}{qs} \frac{1}{3} \times \sum_{\mu SM} |\langle \psi_d^\mu | H^\pi | \phi^{SM} \rangle|^2, \quad (5)$$

is related to its inverse by detailed balance, i.e. by the factor $\frac{4}{3}p^2/q^2$. Here the integrated absorption cross section needs an additional factor $\frac{1}{2}$ from the identity of the protons in

the case of positive pion absorption. The np reactions need the same factor $\frac{1}{2}$ for the isospin already in the differential cross section, so the two total absorption cross sections should be equal for exact isospin symmetry. A similar factorization as above is possible also here with the same quantities R , but different phase space factors. Of course, the inverse of (2) is hardly experimentally feasible, but we keep it for symmetry. Often it is better to present the cross sections in terms of σ_{abs} and Fig. 1 facilitates a comparison in this presentation.

Both the phase space factor (P) and the sum of the squared matrix elements (R) depend on the charge via masses and different momenta. One sees easily the following relation for the difference of the production cross sections (both differential and total)

$$\delta\sigma \equiv 2\sigma(np) - \sigma(pp) = \frac{P_{np} + P_{pp}}{2}(R_{np} - R_{pp}) + (P_{np} - P_{pp})\frac{R_{np} + R_{pp}}{2}. \quad (6)$$

A similar equality holds for the differential absorption cross sections with just different phase space factors P . (For the total absorption cross section difference one should use $\sigma_{\text{abs}}(np) - \sigma_{\text{abs}}(pp)$.) Also the sum of the cross sections can be expressed exactly as

$$2\sigma_{\text{av}} \equiv 2\sigma(np) + \sigma(pp) = \frac{P_{np} + P_{pp}}{2}(R_{np} + R_{pp}) + (P_{np} - P_{pp})\frac{R_{np} - R_{pp}}{2}, \quad (7)$$

where the latter term is only a high order correction. Neglecting it as a good approximation one gets finally the relative change of the cross section simply as the sum of the relative changes of the phase space factor and of the squared matrix elements

$$\frac{\delta\sigma}{\sigma_{\text{av}}} \approx \frac{\delta R}{R_{\text{av}}} + \frac{\delta P}{P_{\text{av}}} \quad (8)$$

with P_{av} and R_{av} average quantities between the two reactions.

From the above discussion it seems most reasonable to present the theoretical results in terms of the calculated ratio $\delta R/R_{\text{av}}$ with

$$\delta R = R_{np} - R_{pp} = \sum_{\mu SM} 2 |\langle \psi_d^\mu | H^\pi | \phi^{SM} \rangle_{np}|^2 - |\langle \psi_d^\mu | H^\pi | \phi^{SM} \rangle_{pp}|^2, \quad (9)$$

which can be used above to estimate the relative difference in the cross sections. Replacing the average σ_{av} by either σ_{pp} or σ_{np} one can then get an estimate of the other cross section with a very small error. This prescription presumably minimizes the systematic effects of possible normalization deviations of the model calculation from experiments, since if the cross section is overestimated, likely also the difference is. This expectation will be borne out later in the discussion of the model dependence. Also the choice of the placement of the constant statistical factors either in P or R now actually becomes irrelevant in these relative quantities.

Fig. 1 shows $\delta R/R_{\text{av}}$ and $\delta P/P_{\text{av}}$ in different cases as a function of $\eta = q/m_{\pi^+}$. First, the dashed curve shows only the mass difference effect. At threshold with delicate cancellations of the oscillatory NN wave function, the long-ranged Galilean invariance

term $\propto (\vec{p} + \vec{p}') \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)$ [10] yields a larger amplitude for the reaction (2). Although this is only a quarter of the dominant s -wave rescattering amplitude, this source is still responsible for about +2 percent units in $\delta R/R_{av}$. Further, the pp momentum for the same pion momentum close to threshold is about two percent larger than the np momentum. Because of the wave function normalization as $\sim j_L(pr + \delta_L)$ this makes the cross section of the reaction (1) 4 % smaller than (2). The wide and deep dip in Fig. 1 is due to the Δ isobar, which increases both cross sections. However, for any given pion momentum the necessary pp kinetic energy is higher and, therefore, closer to the resonance as long as the energy is still below the Δ threshold. (At threshold the difference in the distance to the Δ is about 5 %.) Therefore, the reaction (1) becomes stronger than (2) just below the Δ region, while the effect in $\delta\sigma$ is reversed above it, where the pp energy is further from the Δ . Here it is essential to remember that the distance of the ΔN mass from the two-nucleon mass is the same for both np and pp channels as discussed above before Eq. (4). This strong energy dependence of the cross section difference may be somewhat unexpected but perfectly understandable. In size the mass effect alone is, in fact, already comparable to today's best experimental uncertainties.

However, in $pp \rightarrow d\pi^+$ there is also the Coulomb force present. Although for the high-energy initial state this can arguably be neglected, it is important in the final state close to threshold. It may be noted that for larger pion energies both Coulomb effects become comparable, since the pp reduced mass is much larger than the pion mass. However, the Coulomb effect in the initial pp state is always smooth and rather small and we do not consider it here. The two become about equal in the neighbourhood of $\eta \approx 1.2$.

In this work we consider the Coulomb effect in three different ways. First we take it into account in different pion partial waves by the Coulomb penetration factors [12]

$$C_l(\xi) = \left[\prod_{n=1}^l \left(1 + \frac{\xi^2}{n^2} \right) \right]^{\frac{1}{2}} C_0(\xi). \quad (10)$$

Here the denominator $(2l+1)!!$ present also in the series expansion of the spherical Bessel functions has been removed for a direct comparison with plane waves. As can be seen in e.g. the Appendix of the first of Refs. [10], for plane wave pions some spherical Bessel functions $j_l(qr/2)$ enter the overlap integrals, where l is not necessarily the same as the pion angular momentum l_π relative to the deuteron. We multiply these by the above $C_l(\xi)$. As $\xi = \alpha m_{red} c^2 / (\hbar c q) \approx 0.0068/\eta$ is rather small for any presently reported value of η , all penetration factors are practically the same as $C_0(\xi) = [2\pi\xi(e^{2\pi\xi} - 1)^{-1}]^{1/2}$ (m_{red} is the πd reduced mass). The difference with the inclusion also of this effect is shown by the solid curve in Fig. 1. Below the Δ resonance the negative difference is partly cancelled off because of the Coulomb suppression of the $d\pi^+$ final state, while at threshold $\delta R/R_{av}$ approaches 2.

Also for completeness, the dotted and dash-dot curves show the relative change of the phase space factors $\delta P/P_{av}$ for the production and absorption reactions, respectively. The relative cross section difference $\delta\sigma/\sigma_{av}$ can now be obtained simply by adding to $\delta R/R_{av}$ the corresponding $\delta P/P_{av}$, as shown in Eq. 8.

A relatively standard way of including Coulomb corrections has been to apply to the cross section the penetration factor C_0^2 . Using only this gave results indistinguishable from the procedure described above. Also the use of C_{l_π} in various pion amplitudes directly did not deviate significantly. Another way (though still not exact) of including the Coulomb effect is to explicitly distort the Bessel functions to the Coulomb functions $j_l \rightarrow F_l$. However, one should note that this has some difficulty as having different cases: either an emission of a charged pion with also a charged nucleon (as $\Delta^{++} \rightarrow p\pi^+$) or a proton or a Δ^+ producing a π^+ , which interacts by the Coulomb force (possibly combined with s-wave rescattering) with the *second* nucleon. A reasonable estimate may be obtained by a simple replacement of the wave functions which should maximize the effect of this change. Only in case of serious disagreement with the earlier result is there reason for worry. However, the results obtained in this way were essentially the same as those with the simpler multiplicative penetration factors.

A third way of looking at the Coulomb effects is to consider also the finite size of the charge distribution of the deuteron. For this purpose we have modified the Coulomb potential due to the charge distribution of the Reid deuteron wave function to

$$V_{\text{Coul}}^m(\vec{r}_\pi) = e^2 \int d\Omega dr \frac{u^2(r) + v^2(r)}{|\vec{r}_\pi - \vec{r}/2|} \quad (11)$$

and used it in a Schrödinger equation to numerically solve the relevant pion wave functions to be used in the matrix elements. This approach neglects the nonspherical parts of the potential due to the D state, which would cause a coupling between different pion partial waves. The effect of charge extension is to very slightly increase the penetration (i.e. weaken the repulsion), but contrary to the expectation of Ref. [13] the change is not large enough at any energy to warrant an additional calculation of the observables (at the level 1.5% for s waves and 0.5% for p waves). Table I shows the penetration factors C_0^2 and C_1^2 for a point-like and extended charge at a few low momenta. In the threshold parametrization $\sigma_{\text{prod}} = \alpha\eta + \beta\eta^3$ these have been used in the past to scale the parameters α and β . Even though the deuteron is rather extended by nuclear scale, its spatial variation is much faster than that of the pion wave function at very low energies. So basically threshold pions then feel a point charge. Further above threshold the Coulomb force loses importance. So either way the influence of the charge distribution is rather negligible to the reaction. This insensitivity to the details of the charge distribution also suggests that the effect of deuteron polarizability can be rather safely neglected. It may be noted that the present finite-size effects are only about a quarter of the results of Ref. [6], which, integrating only to r_π , missed part of the charge [14].

Fig. 2 shows the corresponding changes in the analyzing power A_y at 90° for a polarized beam, which is rather representative, since its angular shape changes slowly and systematically. The effect of the mass difference alone is rather small, as can be seen from the dashed curve. The result remains indistinguishable from this, if also the Coulomb penetration factors are applied on the pion wave functions. This means, of course, that the effects are similar in different partial waves and do not change relative

phases. However, adding the asymptotic Coulomb phase σ_{l_π} to the amplitudes does change the relative phases and has a significant effect below $\eta \approx 0.8$ as can be seen from the solid curve. To emphasize the origin of this effect the dotted curve shows still the difference $A_y(\sigma_{l_\pi} = 0) - A_y(\sigma_{l_\pi} \neq 0)$ for the pure pp case; clearly the Coulomb phases σ_{l_π} dominate the change of A_y .

It was argued above that the chosen presentation of the results as the ratio $\delta R/R_{\text{ave}}$ would minimize the role of theoretical uncertainties and model dependence. We tested this in various ways. Using the Paris deuteron wave function instead of the Reid soft core one, threshold production ratio changes by about +0.6% and δA_y by +0.01. In the Paris wave function there is a significantly softer hard core. At higher energies the changes are smaller.

Next the initial state interaction was changed strongly by enhancing the $NN \rightarrow N\Delta$ transition potential by 20%. In the pion production energy region this changes the NN phase shifts by several degrees and the cross section is changed by 10-25%. Nevertheless, at threshold the effects in both $\delta R/R_{\text{ave}}$ and δA_y were totally negligible. In the Δ region the change in the ratio $\delta R/R_{\text{ave}}$ was about -1% and in δA_y -0.005. Such a large variation of the transition potential is, in fact, not allowed. Namely, in Ref. [15] a strong point was made that the Δ strength can be well fixed by the total pion production cross section in the Δ region, so there is not much uncertainty from this source. As an ultimate test the Δ was switched off and only the standard Reid soft core NN potential used. At threshold this has an effect of reducing the cross section by a factor of $\frac{1}{3}$ [11]. Even with this massive overall change the threshold value of the $\delta R/R_{\text{ave}}$ increased by only 2 percent units to 9%, and then this quantity decreases monotonously from this value with increasing energy being about 3% at $\eta = 1.8$. This gives a measure of the relative overall importance of the Δ at threshold. At higher energies its role is more pronounced, as testified by the strong dip, which exclusively arises from approaching and passing the resonance. Here the Δ effect is about as striking as in the total cross section itself. However, as discussed above, there is little uncertainty in the role of the Δ . On the other hand, alterations of only the nucleon correlations have much less effect as was seen e.g. in Ref. [15].

Recently arguments have arisen about the importance of off-shell rescattering of the pion. Presently we consider only a monopole form factor in the whole rescattering process with the πN scattering amplitude taken from the on-shell analysis of Ref. [16], which assumes charge independence. While the form factor should be reasonable for the isovector scattering presumably mediated by ρ^\pm mesons, it may not be sufficient for the isospin symmetric isoscalar channel. Varying the cut-off mass by a few hundred MeV had very little effect on the difference between the two reactions, while the cross section itself changed by tens of percent at threshold. However, at a deeper level the different masses of the intermediate mesons could have some charge dependent effect in the off-shell scattering amplitude, but such a calculation would require an actual model of πN scattering, based perhaps on chiral perturbation theory. One would expect some effect also from the different pion masses in the intermediate pion propagator. However, both

of these effects belong to the class of dynamic isotensor forces outside the scope of this Letter. Work on such interactions is in progress.

In summary, we have studied the changes in the total cross sections and analyzing powers $A_y(90^\circ)$ of the reactions $pp \rightarrow d\pi^+$ and $np \rightarrow d\pi^0$ caused by the "trivial" differences in the masses and the Coulomb interaction. Depending on energy, these can be at a several percent level with varying sign, and in detailed comparisons of data from different reactions they should be taken into account as well as in calibrating neutron beams by pion production. While the Coulomb effect can be treated simply by multiplicative factors $C_0(\xi)^2$, the mass difference causes a nontrivially energy dependent effect, which requires an actual model calculation, the task accomplished here. The angular distributions change significantly less than the above observables setting an overall scale of the changes. Overall, our estimation of the uncertainties of the present calculation (within its scope to consider only the mass differences in kinematics and the Coulomb effect) would be about one percent unit for the $\delta R/R_{\text{ave}}$ and 0.01 for δA_y .

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TABLE

Table 1: The penetration factors for a point-like charge and an extended distribution (labelled e).

η	C_0^2	C_0^{e2}	C_1^2	C_1^{e2}
0.01	0.061	0.061	0.089	0.089
0.02	0.286	0.291	0.319	0.321
0.04	0.560	0.568	0.576	0.579
0.06	0.686	0.696	0.695	0.698
0.08	0.757	0.768	0.762	0.766
0.10	0.802	0.813	0.805	0.809
0.15	0.864	0.877	0.866	0.871
0.20	0.897	0.910	0.898	0.903
0.30	0.931	0.943	0.931	0.936

FIGURE CAPTIONS

Fig. 1: Relative changes in the sum of the squared matrix elements (R) and phase space factors (P). Dashed curve: the mass difference effect in R ; solid: the full result for R including also the Coulomb repulsion; dotted: the change in P for production; dash-dot: the change in P for absorption.

Fig. 2: Differences between $\vec{n}p \rightarrow d\pi^0$ and $\vec{p}p \rightarrow d\pi^+$ in the analyzing power A_y at 90° . Dotted: the mass difference effect alone (indistinguishable if also the penetration factors are included); solid: the Coulomb phase σ_{l_π} included; dashed: the resulting change in the $\vec{p}p \rightarrow d\pi^+$ reaction, if the Coulomb phases are switched off.